

Quiz 20

April 22, 2016

1. Find the inverse of the following matrix:

$$\begin{bmatrix} 2 & 0 & -6 \\ -1 & -2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & -6 & 1 & 0 & 0 \\ -1 & -2 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{1}{2} & 0 & 0 \\ -1 & -2 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_1+R_2 \rightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{1}{2} & 0 & 0 \\ 0 & -2 & -1 & \frac{1}{2} & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{-R_2+R_3 \rightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & 1 \end{array} \right]$$

$$\xrightarrow{2R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 1 & 2 \end{array} \right]$$

$$\xrightarrow{\begin{array}{l} 3R_3+R_1 \rightarrow R_1 \\ -\frac{1}{2}R_3+R_2 \rightarrow R_2 \end{array}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 3 & 6 \\ 0 & 1 & 0 & -\frac{1}{2} & -1 & -1 \\ 0 & 0 & 1 & \frac{1}{2} & 1 & 2 \end{array} \right]$$

Check: $\begin{bmatrix} 2 & 0 & -6 \\ -1 & -2 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 6 \\ -\frac{1}{2} & -1 & -1 \\ \frac{1}{2} & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$

2. The inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & -1 & 1 \\ 2 & 3 & 6 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} -3 & 0 & 2 \\ 8/3 & -2/3 & -5/3 \\ -1/3 & 1/3 & 1/3 \end{bmatrix}$.

(a) Find the value of y if
$$\begin{cases} x + 2y + 4z = 3 \\ -x - y + z = -6 \\ 2x + 3y + 6z = 9 \end{cases}$$

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$$\begin{bmatrix} 1 & 2 & 4 \\ -1 & -1 & 1 \\ 2 & 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -6 \\ 9 \end{bmatrix}, \text{ so } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} 3 \\ -6 \\ 9 \end{bmatrix}.$$

$$y = \frac{8}{3}(3) + \left(-\frac{2}{3}\right)(-6) + \left(-\frac{5}{3}\right)(9) \\ = 8 + 4 - 15 = \boxed{-3}$$

(b) Find the value of x if
$$\begin{cases} x + 2y + 4z = 1 \\ -x - y + z = -15 \\ 2x + 3y + 6z = 0 \end{cases}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} 1 \\ -15 \\ 0 \end{bmatrix}, \text{ so } x = -3(1) + 0(-15) + 2(0) = \boxed{-3}$$